Iterative Processes of Algebraic Closures, Completions, and Y_m Number Systems: An Infinite Landscape of Fields and Rings

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Abstract

This paper explores the iterative processes involving algebraic closures, Archimedean and non-Archimedean completions, Y_m number systems, and Fontaine's rings. By systematically applying combinations of these transformations, we generate an infinite number of distinct fields and rings. Each iteration introduces new algebraic structures and symmetries, reflected in the complex Galois groups of the resulting extensions. We examine the properties of these fields through detailed examples for increasing n, the number of iterations, and analyze the impact of mixed Archimedean and non-Archimedean completions. This approach provides deep insights into the rich landscape of advanced mathematical structures and their automorphisms.

Iterative Processes with Mixed Completions and Closures

We consider the iterative processes involving algebraic closures, Archimedean and non-Archimedean completions, Y_m number systems, and Fontaine's rings. The notation will be as follows:

- (AC): Taking the algebraic closure.
- $[C]_A$: Taking the Archimedean completion.
- $[C]_N$: Taking the non-Archimedean completion (e.g., p-adic completion).
- $[Y_m]$: Applying the Y_m number system transformation.
- $[F]$: Applying transformations using Fontaine's rings.

1 For $n = 3$

Process: $(AC)_3[Y_m][F][C]_N[C]_A[C]_N(\mathbb{Q})$

Step-by-Step Breakdown:

1. $[C]_N(\mathbb{Q}) = \mathbb{Q}_p$
   - Non-Archimedean completion of $\mathbb{Q}$ at a prime p.
   - Galois group: $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$, the group of automorphisms of the algebraic closure of $\mathbb{Q}_p$ over $\mathbb{Q}_p$, which is a profinite group capturing all possible symmetries of $\overline{\mathbb{Q}_p}$.
2. $[C]_A(Q_p) = \mathbb{C}_p$
   - Archimedean completion of $\mathbb{Q}_p$.
   - Galois group: $\text{Gal}(\mathbb{C}_p/\mathbb{Q}_p)$, generally trivial, as $\mathbb{C}_p$ is complete and algebraically closed.
3. $[C]_N(\mathbb{C}_p) = \mathbb{C}_p$
   - Non-Archimedean completion does not change $\mathbb{C}_p$.
   - Galois group: Trivial since the field remains the same.
4. $[F](\mathbb{C}_p) = \mathbb{A}_{inf}(\mathbb{C}_p)$
   - Applying Fontaine's ring to $\mathbb{C}_p$.
   - Galois group: The Galois group of $\mathbb{A}_{inf}$ over $\mathbb{Q}_p$ involves the p-adic Hodge structures, including intricate relations with $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$.
5. $[Y_m](\mathbb{A}_{inf}(\mathbb{C}_p)) = Y_m(\mathbb{A}_{inf}(\mathbb{C}_p))$
   - Applying Y_m number system transformation.
   - Galois group: Depends on Y_m structure, adding another layer of automorphisms specific to the Y_m system.
6. $(AC)(Y_m(\mathbb{A}_{inf}(\mathbb{C}_p))) = \overline{Y_m(\mathbb{A}_{inf}(\mathbb{C}_p))}$
   - Taking the algebraic closure.
   - Galois group: $\text{Gal}(\overline{Y_m(\mathbb{A}_{inf}(\mathbb{C}_p))}/Y_m(\mathbb{A}_{inf}(\mathbb{C}_p)))$, capturing all symmetries of the algebraic closure of $Y_m(\mathbb{A}_{inf}(\mathbb{C}_p))$.
7. Repeated algebraic closures.
   - Galois group: Reflects iterated algebraic closures, each step potentially introducing new symmetries.

2 For $n = 4$

Process: $(AC)_4[Y_m][F][C]_N[C]_A[C]_N[Y_m][F](\mathbb{Q})$

Step-by-Step Breakdown:

1. $[C]_N(\mathbb{Q}) = \mathbb{Q}_p$
   - Non-Archimedean completion of $\mathbb{Q}$.
   - Galois group: $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$, the profinite group governing the automorphisms of the algebraic closure of $\mathbb{Q}_p$.
2. $[C]_A(\mathbb{Q}_p) = \mathbb{C}_p$
   - Archimedean completion of $\mathbb{Q}_p$.
   - Galois group: $\text{Gal}(\mathbb{C}_p/\mathbb{Q}_p)$, typically trivial.
3. $[C]_N(\mathbb{C}_p) = \mathbb{C}_p$
   - Non-Archimedean completion does not change $\mathbb{C}_p$.
   - Galois group: Trivial.
4. $[F](\mathbb{C}_p) = \mathbb{A}_{inf}(\mathbb{C}_p)$
   - Applying Fontaine's ring.
   - Galois group: Involves deep p-adic Hodge theory, related to $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$.
5. $[Y_m](\mathbb{A}_{inf}(\mathbb{C}_p)) = Y_m(\mathbb{A}_{inf}(\mathbb{C}_p))$
   - Applying Y_m number system.
   - Galois group: Depends on the dimension and properties of Y_m.
6. $[F](Y_m(A_{inf}(\mathbb{C}_p)))$
   - Applying Fontaine's ring again.
   - Galois group: More intricate structures involving p-adic elements.
7. $[Y_m](\mathbb{A}_{inf}(Y_m(\mathbb{A}_{inf}(\mathbb{C}_p))))$
   - Another Y_m transformation.
   - Galois group: Further complexities depending on Y_m.
8. $(AC)(Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{A}_{inf}(\mathbb{C}_p)))))$
   - Taking the algebraic closure.
   - Galois group: $\text{Gal}(\overline{Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{A}_{inf}(\mathbb{C}_p))))}/Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{A}_{inf}(\mathbb{C}_p)))))$.
9. Repeated algebraic closures.

3 For $n = 5$

Process: $(AC)_5[Y_m][F][C]_N[C]_A[C]_N[Y_m][F][C]_A(\mathbb{Q})$

Step-by-Step Breakdown:

1. $[C]_N(\mathbb{Q}) = \mathbb{Q}_p$
   - Non-Archimedean completion.
   - Galois group: $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$.
2. $[C]_A(\mathbb{Q}_p) = \mathbb{C}_p$
   - Archimedean completion.
   - Galois group: $\text{Gal}(\mathbb{C}_p/\mathbb{Q}_p)$.
3. $[C]_N(\mathbb{C}_p) = \mathbb{C}_p$
   - Non-Archimedean completion.
   - Galois group: Trivial.
4. $[F](\mathbb{C}_p) = \mathbb{A}_{inf}(\mathbb{C}_p)$
   - Fontaine's ring.
   - Galois group: Related to p-adic Hodge theory.
5. $[Y_m](\mathbb{A}_{inf}(\mathbb{C}_p)) = Y_m(\mathbb{A}_{inf}(\mathbb{C}_p))$
   - Y_m number system.
   - Galois group: Depends on Y_m.
6. $[F](Y_m(\mathbb{A}_{inf}(\mathbb{C}_p)))$
   - Fontaine's ring.
   - Galois group: Further p-adic structures.
7. $[Y_m](\mathbb{A}_{inf}(Y_m(\mathbb{A}_{inf}(\mathbb{C}_p))))$
   - Another Y_m transformation.
   - Galois group: Complex, depending on Y_m.
8. $[C]_A(Y_m(\mathbb{A}_{inf}(Y_m(A_{inf}(\mathbb{C}_p))))) = \mathbb{C}$
   - Archimedean completion stabilizes at $\mathbb{C}$.
   - Galois group: Trivial.
9. $(AC)(\mathbb{C}) = \mathbb{C}$
   - $\mathbb{C}$ is already algebraically closed.
   - Galois group: Trivial.
10. Repeated algebraic closures, remaining at $\mathbb{C}$.

4 Galois Groups and Infinite Possibilities

Each iterative process introduces new algebraic structures and symmetries, reflected in increasingly complex Galois groups. Mixed Archimedean and non-Archimedean completions combined with Y_m transformations and Fontaine's rings lead to highly intricate field extensions. By varying m, k, l, j and their order, an infinite number of distinct fields and rings can be generated, each with unique Galois groups.

5 Further Increasing n

5.1 For $n = 6$

Process: $(AC)_6[Y_m][F][C]_N[C]_A[Y_m][F][C]_N[Y_m][F][C]_A(\mathbb{Q})$

Step-by-Step Breakdown:

1. $[C]_N(\mathbb{Q}) = Q_p$
   - Non-Archimedean completion.
   - Galois group: $\mathrm{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$.
2. $[C]_A(\mathbb{Q}_p) = \mathbb{C}_p$
   - Archimedean completion.
   - Galois group: $\mathrm{Gal}(\mathbb{C}_p/\mathbb{Q}_p)$.
3. $[Y_m](\mathbb{C}_p) = Y_m(\mathbb{C}_p)$
   - Y_m number system.
   - Galois group: Depends on Y_m.
4. $[F](Y_m(\mathbb{C}_p)) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$
   - Fontaine's ring.
   - Galois group: Related to p-adic Hodge theory.
5. $[C]_N(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$
   - Non-Archimedean completion.
   - Galois group: Trivial.
6. $[Y_m](\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p)))$
   - Another Y_m transformation.
   - Galois group: Complex, depending on Y_m.
7. $[F](Y_m(\mathbb{A}_{inf}(Y_m(C_p))))$
   - Fontaine's ring.

- Galois group: Further p-adic structures.

8. $[C]_A(Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))))) = \mathbb{C}$
   - Archimedean completion stabilizes at $\mathbb{C}$.
   - Galois group: Trivial.
9. $(AC)(\mathbb{C}) = \mathbb{C}$
   - C is already algebraically closed.
   - Galois group: Trivial.
10. Repeated algebraic closures, remaining at $\mathbb{C}$.

5.2 For $n = 7$

Process: $(AC)_7[Y_m][F][C]_N[C]_A[Y_m][F][C]_N[Y_m][F][C]_A[Y_m][F](\mathbb{Q})$
Step-by-Step Breakdown:

1. $[C]_N(\mathbb{Q}) = \mathbb{Q}_p$
   - Non-Archimedean completion.
   - Galois group: Gal$(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$.
2. $[C]_A(\mathbb{Q}_p) = \mathbb{C}_p$
   - Archimedean completion.
   - Galois group: Gal$(\mathbb{C}_p/\mathbb{Q}_p)$.
3. $[Y_m](\mathbb{C}_p) = Y_m(\mathbb{C}_p)$
   - Y_m number system.
   - Galois group: Depends on Y_m.
4. $[F](Y_m(\mathbb{C}_p)) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$
   - Fontaine's ring.
   - Galois group: Related to p-adic Hodge theory.
5. $[C]_N(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = \mathbb{A}_{inf}(Y_m(C_p))$
   - Non-Archimedean completion.
   - Galois group: Trivial.
6. $[Y_m](\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p)))$
   - Another Y_m transformation.
   - Galois group: Complex, depending on Y_m.
7. $[F](Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))))$

- Fontaine's ring.
- Galois group: Further p-adic structures.

8. $[C]_A(Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))))) = \mathbb{C}$
    - Archimedean completion stabilizes at $\mathbb{C}$.
    - Galois group: Trivial.
9. $(AC)(\mathbb{C}) = \mathbb{C}$
    - $\mathbb{C}$ is already algebraically closed.
    - Galois group: Trivial.
10. Repeated algebraic closures, remaining at $\mathbb{C}$.

5.3 For $n = 8$

Process: $(AC)_8[Y_m][F][C]_N[C]_A[Y_m][F][C]_N[Y_m][F][C]_A[Y_m][F][C]_N(\mathbb{Q})$
Step-by-Step Breakdown:

1. $[C]_N(\mathbb{Q}) = \mathbb{Q}_p$
    - Non-Archimedean completion.
    - Galois group: $\mathrm{Gal}(\overline{\mathbb{Q}_p}/Q_p)$.
2. $[C]_A(\mathbb{Q}_p) = \mathbb{C}_p$
    - Archimedean completion.
    - Galois group: $\mathrm{Gal}(\mathbb{C}_p/\mathbb{Q}_p)$.
3. $[Y_m](\mathbb{C}_p) = Y_m(\mathbb{C}_p)$
    - Y_m number system.
    - Galois group: Depends on Y_m.
4. $[F](Y_m(\mathbb{C}_p)) = \mathbb{A}_{inf}(Y_m(C_p))$
    - Fontaine's ring.
    - Galois group: Related to p-adic Hodge theory.
5. $[C]_N(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = \mathbb{A}_{inf}(Y_m(C_p))$
    - Non-Archimedean completion.
    - Galois group: Trivial.
6. $[Y_m](A_{inf}(Y_m(\mathbb{C}_p))) = Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p)))$
    - Another Y_m transformation.
    - Galois group: Complex, depending on Y_m.

7. $[F](Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))))$
    - Fontaine's ring.
    - Galois group: Further p-adic structures.
8. $[C]_A(Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))))) = \mathbb{C}$
    - Archimedean completion stabilizes at $\mathbb{C}$.
    - Galois group: Trivial.
9. $(AC)(\mathbb{C}) = \mathbb{C}$
    - C is already algebraically closed.
    - Galois group: Trivial.
10. Repeated algebraic closures, remaining at $\mathbb{C}$.

5.4 For $n = 9$

Process: $(AC)_9[Y_m][F][C]_N[C]_A[Y_m][F][C]_N[Y_m][F][C]_A[Y_m][F][C]_N[Y_m][F](\mathbb{Q})$
Step-by-Step Breakdown:

1. $[C]_N(\mathbb{Q}) = \mathbb{Q}_p$
    - Non-Archimedean completion.
    - Galois group: $\text{Gal}(\overline{\mathbb{Q}_p}/Q_p)$.
2. $[C]_A(\mathbb{Q}_p) = C_p$
    - Archimedean completion.
    - Galois group: $\text{Gal}(\mathbb{C}_p/Q_p)$.
3. $[Y_m](C_p) = Y_m(\mathbb{C}_p)$
    - Y_m number system.
    - Galois group: Depends on Y_m.
4. $[F](Y_m(\mathbb{C}_p)) = A_{inf}(Y_m(\mathbb{C}_p))$
    - Fontaine's ring.
    - Galois group: Related to p-adic Hodge theory.
5. $[C]_N(A_{inf}(Y_m(\mathbb{C}_p))) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$
    - Non-Archimedean completion.
    - Galois group: Trivial.
6. $[Y_m](\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = Y_m(\mathbb{A}_{inf}(Y_m(C_p)))$
    - Another Y_m transformation.

- Galois group: Complex, depending on Y_m.

7. $[F](Y_{m(\mathbb{A}_{inf}(Y_m(C_p))))})$
   - Fontaine's ring.
   - Galois group: Further p-adic structures.
8. $[C]_A(Y_m(A_{inf}(Y_m(C_p))))) = \mathbb{C}$
   - Archimedean completion stabilizes at C.
   - Galois group: Trivial.
9. $(AC)(\mathbb{C}) = \mathbb{C}$
   - $\mathbb{C}$ is already algebraically closed.
   - Galois group: Trivial.
10. Repeated algebraic closures, remaining at $\mathbb{C}$.

5.5 For $n = 10$

Process: $(AC)_{10}[Y_m][F][C]_N[C]_A[Y_m][F][C]_N[Y_m][F][C]_A[Y_m][F][C]_N[Y_m][F](\mathbb{Q})$
Step-by-Step Breakdown:

1. $[C]_N(Q) = Q_p$
   - Non-Archimedean completion.
   - Galois group: Gal($\overline{\mathbb{Q}_p}/\mathbb{Q}_p$).
2. $[C]_A(\mathbb{Q}_p) = \mathbb{C}_p$
   - Archimedean completion.
   - Galois group: Gal($\mathbb{C}_p/Q_p$).
3. $[Y_m](\mathbb{C}_p) = Y_m(C_p)$
   - Y_m number system.
   - Galois group: Depends on Y_m.
4. $[F](Y_m(\mathbb{C}_p)) = A_{inf}(Y_m(C_p))$
   - Fontaine's ring.
   - Galois group: Related to p-adic Hodge theory.
5. $[C]_N(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = A_{inf}(Y_m(\mathbb{C}_p))$
   - Non-Archimedean completion.
   - Galois group: Trivial.
6. $[Y_m](\mathbb{A}_{inf}(Y_m(C_p))) = Y_m(\mathbb{A}_{inf}(Y_m(C_p)))$

- Another Y_m transformation.
- Galois group: Complex, depending on Y_m.

7. $[F](Y_m(A_{inf}(Y_m(C_p))))$
   - Fontaine's ring.
   - Galois group: Further p-adic structures.
8. $[C]_A(Y_m(A_{inf}(Y_m(C_p))))) = \mathbb{C}$
   - Archimedean completion stabilizes at C.
   - Galois group: Trivial.
9. $(AC)(\mathbb{C}) = C$
   - C is already algebraically closed.
   - Galois group: Trivial.
10. Repeated algebraic closures, remaining at C.

5.6 For $n = 11$

Process: $(AC)_{11}[Y_m][F][C]_N[C]_A[Y_m][F][C]_N[Y_m][F][C]_A[Y_m][F][C]_N[Y_m][F](\mathbb{Q})$
Step-by-Step Breakdown:

1. $[C]_N(\mathbb{Q}) = \mathbb{Q}_p$
   - Non-Archimedean completion.
   - Galois group: $\mathrm{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$.
2. $[C]_A(\mathbb{Q}_p) = \mathbb{C}_p$
   - Archimedean completion.
   - Galois group: $\mathrm{Gal}(\mathbb{C}_p/\mathbb{Q}_p)$.
3. $[Y_m](\mathbb{C}_p) = Y_m(\mathbb{C}_p)$
   - Y_m number system.
   - Galois group: Depends on Y_m.
4. $[F](Y_m(\mathbb{C}_p)) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$
   - Fontaine's ring.
   - Galois group: Related to p-adic Hodge theory.
5. $[C]_N(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$
   - Non-Archimedean completion.
   - Galois group: Trivial.

6. $[Y_m](\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = Y_m(A_{inf}(Y_m(\mathbb{C}_p)))$
    - Another Y_m transformation.
    - Galois group: Complex, depending on Y_m.
7. $[F](Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))))$
    - Fontaine's ring.
    - Galois group: Further p-adic structures.
8. $[C]_A(Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))))) = \mathbb{C}$
    - Archimedean completion stabilizes at $\mathbb{C}$.
    - Galois group: Trivial.
9. $(AC)(\mathbb{C}) = \mathbb{C}$
    - $\mathbb{C}$ is already algebraically closed.
    - Galois group: Trivial.
10. Repeated algebraic closures, remaining at $\mathbb{C}$.

5.7 For $n = 12$

Process: $(AC)_{12}[Y_m][F][C]_N[C]_A[Y_m][F][C]_N[Y_m][F][C]_A[Y_m][F][C]_N[Y_m][F](\mathbb{Q})$
Step-by-Step Breakdown:

1. $[C]_N(\mathbb{Q}) = \mathbb{Q}_p$
    - Non-Archimedean completion.
    - Galois group: $\mathrm{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$.
2. $[C]_A(\mathbb{Q}_p) = \mathbb{C}_p$
    - Archimedean completion.
    - Galois group: $\mathrm{Gal}(\mathbb{C}_p/\mathbb{Q}_p)$.
3. $[Y_m](\mathbb{C}_p) = Y_m(\mathbb{C}_p)$
    - Y_m number system.
    - Galois group: Depends on Y_m.
4. $[F](Y_m(\mathbb{C}_p)) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$
    - Fontaine's ring.
    - Galois group: Related to p-adic Hodge theory.
5. $[C]_N(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$
    - Non-Archimedean completion.

- Galois group: Trivial.

6. $[Y_m](\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p)))$
   - Another Y_m transformation.
   - Galois group: Complex, depending on Y_m.
7. $[F](Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))))$
   - Fontaine's ring.
   - Galois group: Further p-adic structures.
8. $[C]_A(Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))))) = \mathbb{C}$
   - Archimedean completion stabilizes at $\mathbb{C}$.
   - Galois group: Trivial.
9. $(AC)(\mathbb{C}) = \mathbb{C}$
   - $\mathbb{C}$ is already algebraically closed.
   - Galois group: Trivial.
10. Repeated algebraic closures, remaining at $\mathbb{C}$.

5.8 For $n = 13$

Process: $(AC)_{13}[Y_m][F][C]_N[C]_A[Y_m][F][C]_N[Y_m][F][C]_A[Y_m][F][C]_N[Y_m][F](\mathbb{Q})$
Step-by-Step Breakdown:

1. $[C]_N(\mathbb{Q}) = \mathbb{Q}_p$
   - Non-Archimedean completion.
   - Galois group: $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$.
2. $[C]_A(\mathbb{Q}_p) = \mathbb{C}_p$
   - Archimedean completion.
   - Galois group: $\text{Gal}(\mathbb{C}_p/\mathbb{Q}_p)$.
3. $[Y_m](\mathbb{C}_p) = Y_m(\mathbb{C}_p)$
   - Y_m number system.
   - Galois group: Depends on Y_m.
4. $[F](Y_m(\mathbb{C}_p)) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$
   - Fontaine's ring.
   - Galois group: Related to p-adic Hodge theory.
5. $[C]_N(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$

- Non-Archimedean completion.
- Galois group: Trivial.

6. $[Y_m](\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p)))$
   - Another Y_m transformation.
   - Galois group: Complex, depending on Y_m.
7. $[F](Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))))$
   - Fontaine's ring.
   - Galois group: Further p-adic structures.
8. $[C]_A(Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))))) = \mathbb{C}$
   - Archimedean completion stabilizes at $\mathbb{C}$.
   - Galois group: Trivial.
9. $(AC)(\mathbb{C}) = \mathbb{C}$
   - $\mathbb{C}$ is already algebraically closed.
   - Galois group: Trivial.
10. Repeated algebraic closures, remaining at $\mathbb{C}$.

5.9 For $n = 14$

Process: $(AC)_{14}[Y_m][F][C]_N[C]_A[Y_m][F][C]_N[Y_m][F][C]_A[Y_m][F][C]_N[Y_m][F](\mathbb{Q})$
Step-by-Step Breakdown:

1. $[C]_N(\mathbb{Q}) = \mathbb{Q}_p$
   - Non-Archimedean completion.
   - Galois group: $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$.
2. $[C]_A(\mathbb{Q}_p) = \mathbb{C}_p$
   - Archimedean completion.
   - Galois group: $\text{Gal}(\mathbb{C}_p/\mathbb{Q}_p)$.
3. $[Y_m](\mathbb{C}_p) = Y_m(\mathbb{C}_p)$
   - Y_m number system.
   - Galois group: Depends on Y_m.
4. $[F](Y_m(\mathbb{C}_p)) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$
   - Fontaine's ring.
   - Galois group: Related to p-adic Hodge theory.

5. $[C]_N(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$
   - Non-Archimedean completion.
   - Galois group: Trivial.
6. $[Y_m](\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p)))$
   - Another Y_m transformation.
   - Galois group: Complex, depending on Y_m.
7. $[F](Y_m(\mathbb{A}_{inf}(Y_m(C_p))))$
   - Fontaine's ring.
   - Galois group: Further p-adic structures.
8. $[C]_A(Y_m(A_{inf}(Y_m(C_p))))) = \mathbb{C}$
   - Archimedean completion stabilizes at $\mathbb{C}$.
   - Galois group: Trivial.
9. $(AC)(\mathbb{C}) = \mathbb{C}$
   - $\mathbb{C}$ is already algebraically closed.
   - Galois group: Trivial.
10. Repeated algebraic closures, remaining at $\mathbb{C}$.

5.10 For $n = 15$

Process: $(AC)_{15}[Y_m][F][C]_N[C]_A[Y_m][F][C]_N[Y_m][F][C]_A[Y_m][F][C]_N[Y_m][F](\mathbb{Q})$
Step-by-Step Breakdown:

1. $[C]_N(\mathbb{Q}) = \mathbb{Q}_p$
   - Non-Archimedean completion.
   - Galois group: $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$.
2. $[C]_A(\mathbb{Q}_p) = \mathbb{C}_p$
   - Archimedean completion.
   - Galois group: $\text{Gal}(\mathbb{C}_p/\mathbb{Q}_p)$.
3. $[Y_m](\mathbb{C}_p) = Y_m(\mathbb{C}_p)$
   - Y_m number system.
   - Galois group: Depends on Y_m.
4. $[F](Y_m(\mathbb{C}_p)) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$
   - Fontaine's ring.

- Galois group: Related to p-adic Hodge theory.

5. $[C]_N(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$
   - Non-Archimedean completion.
   - Galois group: Trivial.
6. $[Y_m](\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p)))$
   - Another Y_m transformation.
   - Galois group: Complex, depending on Y_m.
7. $[F](Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))))$
   - Fontaine's ring.
   - Galois group: Further p-adic structures.
8. $[C]_A(Y_m(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))))) = \mathbb{C}$
   - Archimedean completion stabilizes at $\mathbb{C}$.
   - Galois group: Trivial.
9. $(AC)(\mathbb{C}) = \mathbb{C}$
   - $\mathbb{C}$ is already algebraically closed.
   - Galois group: Trivial.
10. Repeated algebraic closures, remaining at $\mathbb{C}$.

5.11 For $n = 16$

Process: $(AC)_{16}[Y_m][F][C]_N[C]_A[Y_m][F][C]_N[Y_m][F][C]_A[Y_m][F][C]_N[Y_m][F](\mathbb{Q})$
Step-by-Step Breakdown:

1. $[C]_N(\mathbb{Q}) = \mathbb{Q}_p$
   - Non-Archimedean completion.
   - Galois group: $\text{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$.
2. $[C]_A(\mathbb{Q}_p) = \mathbb{C}_p$
   - Archimedean completion.
   - Galois group: $\text{Gal}(\mathbb{C}_p/\mathbb{Q}_p)$.
3. $[Y_m](\mathbb{C}_p) = Y_m(\mathbb{C}_p)$
   - Y_m number system.
   - Galois group: Depends on Y_m.
4. $[F](Y_m(\mathbb{C}_p)) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$

- Fontaine's ring.
- Galois group: Related to p-adic Hodge theory.

5. $[C]_N(\mathbb{A}_{inf}(Y_m(\mathbb{C}_p))) = \mathbb{A}_{inf}(Y_m(\mathbb{C}_p))$
    - Non-Archimedean completion.
    - Galois group: Trivial.